

On Optimal Tournament Contracts with Heterogeneous Agents

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In practice, firms often use tournaments where employees compete for given prizes or the distribution of a fixed amount of bonuses. For example, managers are compensated via relative performance pay (Eriksson (1999)), workers compete in job-promotion tournaments to climb the hierarchy ladder (Baker, Gibbs, and Holmström (1994)), salesmen participate in sales contests (Murphy, Dacin, and Ford (2004)), and workers compete for higher shares in bonus-pool arrangements (Rajan and Reichelstein (2006)). These and similar tournament schemes occur if the relative performance of employees is linked to monetary consequences. Hence, forced-ranking or forced-distribution systems also belong to the class of tournament compensation schemes. Here, supervisors rate their subordinates according to relative performance given a fixed distribution of different grades that can be assigned to the employees (Boyle (2001)).

Three major advantages of corporate tournaments have been highlighted in the literature: (1) tournaments are applicable even in situations where performance information is only ordinal (Lazear and Rosen (1981)). (2) Contrary to individual incentive schemes like bonuses and piece rates, tournaments do work if the

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performance measure is not verifiable to a third party (Malcomson (1984)). This important self-commitment property of tournaments is based on the fact that tournament prizes or bonus pools are fixed in advance and that payment of these prizes is verifiable. Since, the firm must pay the high winner prize to one of the employees, it cannot save labor costs by misrepresenting the performance information. (3) Tournaments filter out common risk so that the firm may save labor costs without harming incentives (Green and Stokey (1983)). In this short paper, I will concentrate on advantages (1) and (2) to characterize optimal tournament contracts for heterogeneous workers under unlimited and limited liability.

The paper is organized as follows. The next section describes the model. In Section 3, I derive the optimal tournament contract for the case where agents are not protected by limited liability. Section 4 deals with the case of limited liability. Section 5 concludes.

The Model

Two risk neutral agents A and B are hired by a risk neutral principal P . The two agents have zero reservation values. The agents with observable abilities a_A and a_B ($a_A \neq a_B$) exert efforts e_A and e_B that lead to monetary output $\sum_{i=1,2} e_i + a_i$ for the principal. Following Gürtler and Kräkel (2010), I assume that P cannot directly observe efforts nor output but receives an unverifiable, ordinal performance signal s about the ranking of the two agents. This signal either equals $s = s_A$ indicating that agent A has performed better than B or $s = s_B$ indicating the opposite. The signal structure can be characterized as follows:¹

$$s = \begin{cases} s_A & \text{if } e_A + a_A - e_B - a_B > \varepsilon \\ s_B & \text{if } e_A + a_A - e_B - a_B < \varepsilon. \end{cases} \quad (1)$$

¹ Note that this kind of additive model has the same structure as the seminal paper by Lazear and Rosen (1981).

Hence, the realization of the relative performance signal depends on the agents' efforts, their abilities and the variable ε that describes an unobservable, exogenous random term (e.g., measurement error) with density g and cdf G . The density g is assumed to be unimodal and symmetric about zero. Intuitively, the higher agent i 's effort choice and/or his ability the more likely the principal will receive the signal $s = s_i$. Based on this relative performance signal, P offers a tournament contract to the agents, consisting of winner and loser prizes. In the following we will differentiate between two cases. If agents are not protected by limited liability, tournament prizes are allowed to be arbitrarily positive or negative. However, if the agents are protected by limited liability, tournament prizes are not allowed to be negative.

Effort e_i entails costs on agent i that are described (in monetary terms) by the function $c(e_i)$ with $c'(e_i), c''(e_i), c'''(e_i) > 0, \forall e_i > 0$, and $c'(0) = c(0) = 0$ ($i = A, B$). To ensure the existence of pure-strategy equilibria in the tournament, I assume that²

$$\sup_{\Delta e, \Delta a} \Delta w \cdot |g'(\Delta e + \Delta a)| < \inf_{e > 0} c''(e) \quad (2)$$

with $\Delta e := e_A - e_B$, $\Delta a := a_A - a_B$ and Δw denoting the spread between winner and loser prize. As a benchmark case, we can compute the efficient or first-best effort level for each agent. This effort maximizes welfare $\sum_{i=1,2} (e_i + a_i - c(e_i))$ and is implicitly described by e^{FB} with $1 = c'(e^{FB})$ for each agent. The timing is the typical one in moral-hazard models. First, the principal offers a contract to each agent. If the agents accept, they will take part in a tournament and simultaneously choose efforts. Then outputs are realized and payments are made.

² For a similar condition see Schöttner (2007). See also Gürtler (2011) on the existence of pure-strategy equilibria.

Unlimited Liability

Let (w_{1i}, w_{2i}) denote the tournament contract that is offered to agent i . According to (1), the winning probabilities of agents A and B are given by $G(e_A - e_B + \Delta a)$ and $1 - G(e_A - e_B + \Delta a)$, respectively, with $\Delta a = a_A - a_B$. The agents maximize

$$EU_A(e_A) = w_{2A} + \Delta w_A G(e_A - e_B + \Delta a) - c(e_A)$$

and

$$EU_B(e_B) = w_{2B} + \Delta w_B [1 - G(e_A - e_B + \Delta a)] - c(e_B)$$

with $\Delta w_i := w_{1i} - w_{2i}$ denoting the prize spread of agent i ($i = A, B$). The equilibrium (e_A^*, e_B^*) is described by the first-order conditions³

$$\Delta w_A g(e_A^* - e_B^* + \Delta a) = c'(e_A^*) \quad (3)$$

$$\text{and } \Delta w_B g(e_A^* - e_B^* + \Delta a) = c'(e_B^*). \quad (4)$$

When designing the optimal tournament contract, P chooses optimal prizes that maximize expected profits subject to the incentive constraints (3) and (4), the participation constraints $EU_i(e_i) \geq 0$ ($i = A, B$) and the principal's self-commitment constraint

$$w_{1A} + w_{2B} = w_{1B} + w_{2A}. \quad (5)$$

Note that without condition (5), P would ex post always claim the winner-loser combination that minimizes the collective wage bill since the signal s is unverifiable. As (5) implies $\Delta w_A = \Delta w_B =: \Delta w$, the equilibrium at the tournament stage will be symmetric. Since the agents are not protected by limited liability,

³ Recall that (2) guarantees existence.

P will choose $\Delta w = 1/g(\Delta a)$ to implement $e_A^* = e_B^* = e^{FB}$, and individualized loser prizes w_{2A} and w_{2B} to extract all rents so that both agents' participation constraints become binding. The following proposition summarizes the findings:⁴

Proposition 1. *Suppose agents are not protected by limited liability. Although agents are heterogeneous, P implements $e_A^* = e_B^* = e^{FB}$ via the optimal tournament contracts (w_{1A}^*, w_{2A}^*) and (w_{1B}^*, w_{2B}^*) with*

$$\begin{aligned} w_{1A}^* &= c(e^{FB}) + \frac{1 - G(\Delta a)}{g(\Delta a)} & \text{and} & & w_{2A}^* &= c(e^{FB}) - \frac{G(\Delta a)}{g(\Delta a)}, \\ w_{1B}^* &= c(e^{FB}) + \frac{G(\Delta a)}{g(\Delta a)} & \text{and} & & w_{2B}^* &= c(e^{FB}) - \frac{1 - G(\Delta a)}{g(\Delta a)}. \end{aligned}$$

The proposition shows that heterogeneity of agents is not a problem even in settings where handicaps are not applicable. Of course, for given tournament prizes equilibrium efforts decrease in the magnitude of agent heterogeneity, Δa , since g has a unique mode at zero, but the optimal prize spread can always be adjusted to induce efficient incentives. The principal is interested in implementing efficient effort since he can extract all efficiency gains via individualized loser prizes that satisfy the self-commitment constraint.

Limited Liability

Under limited liability, we can neglect the agents' participation constraints since each agent has a zero reservation value so that non-negative tournament prizes and $c(0) = 0$ guarantee that the agents cannot do better than accepting any feasible tournament contract. The principal now faces the two limited-liability conditions $w_{2A} \geq 0$ and $w_{2B} \geq 0$. Because of the non-binding participation constraints and the fact that loser prizes decrease incentives and increase implementation costs, the principal optimally chooses $w_{2A}^* = w_{2B}^* = 0$. The self-

⁴ See also Gürtler and Kräkel (2010).

commitment constraint (5) then implies $w_{1A}^* = w_{1B}^* =: w_1^*$. The uniform winner prizes together with the incentive constraints (3) and (4) imply that we have a unique symmetric equilibrium $e_A^* = e_B^* =: e^*$ with $w_1^* g(\Delta a) = c'(e^*)$. The principal thus maximizes

$$2e^* + a_A + a_B - w_1^* = 2e^* + a_A + a_B - \frac{c'(e^*)}{g(\Delta a)}.$$

The first-order condition yields⁵

$$e^* = A(2g(\Delta a)),$$

with $A \equiv c''^{-1}$ denoting the inverse function of c'' , which is monotonically increasing since $c''' > 0$. Altogether, we obtain the following result:

Proposition 2. *If agents are protected by limited liability, P will implement $e_A^* = e_B^* = A(2g(\Delta a))$ by choosing $w_{2A}^* = w_{2B}^* = 0$ and $w_{1A}^* = w_{1B}^* = c'(A(2g(\Delta a)))/g(\Delta a)$.*

Proposition 2 shows that P optimally chooses uniform winner prizes although agents are heterogeneous. This result follows from the limited liability of the agents, which prohibits rent extraction by the principal via negative loser prizes. The best P can do is setting the loser prizes equal to zero. But then uniform winner prizes automatically follow from the self-commitment constraint.

The proposition also shows that implemented effort will no longer be first best due to limited liability as contractual friction. The implemented effort, $A(2g(\Delta a))$, decreases in the degree of agent heterogeneity, Δa , since density g falls to the tails. Intuitively, the more heterogeneous the agents the less intense will be tournament competition so that the principal would have to choose a high winner

⁵ The second-order condition is satisfied since $c''' > 0$.

prize – implying a large rent for the agents – to restore incentives. The principal therefore prefers low-powered incentives under large heterogeneity.

Note that the impact of uncertainty as measured by the variance of the distribution for ε depends on the degree of agent heterogeneity (see Kräkel and Sliwka (2004), Kräkel (2008)). If agent heterogeneity tends to zero (i.e., $\Delta a \rightarrow 0$), then increased uncertainty via a shift of probability mass to the tails and a corresponding decrease of $g(0)$ would unambiguously lead to a decrease of implemented effort. However, under strict agent heterogeneity, tournament competition may be fostered by higher uncertainty. Technically, $g(\Delta a)$ may increase by a shift of probability mass to the tails. The economic intuition is the following. Suppose, there is moderate uncertainty but considerable agent heterogeneity in the initial situation. In that case, equilibrium efforts (for given winner prize) would be quite low since there is a clear favorite and a clear underdog. If now uncertainty increase, the tournament outcome will be less clear, which balances competition. In other words, the underdog may now win by luck which results into higher incentives for both agents.

Conclusion

If the principal uses uniform tournament prizes for both heterogeneous agents, even under unlimited liability the tournament contract will result into inefficiently low effort since the principal has to leave a positive rent to the more able agent. However, if the principal optimally switches to individualized tournament prizes that satisfy the self-commitment constraint, the principal will be able to extract all rents. Consequently, he implements first-best efforts. Hence, Propositions 1 and 2 stress that heterogeneity is not a natural problem for the optimal design of tournaments, but that only the usual contractual frictions from individual incentive schemes (here, the agents' limited liability) apply for tournaments as

well. In the last decade, several extensions of the seminal tournament paper of Lazear and Rosen (1981) have been introduced in the literature. A considerable part of this new literature deals with issues that belong to behavioral economics. For example, Demougin and Fluet (2003) as well as Grund and Sliwka (2005) analyze the impact of inequity aversion in tournaments. In a recent paper, Gill and Stone (2010) investigate the implications of loss aversion for tournament competition. The three papers show that the consequences of the behavioral effects crucially depend on whether tournament prizes are exogenously given (so that only the game between the contestants has to be considered) or endogenously chosen by the principal, who can adjust them optimally to possibly benefit from the behavioral effects.

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