Variance Bounds Tests for the Hypothesis of Efficient Stock Market

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Introduction

The theory of efficient financial markets was regarded as inviolable in academic literature for a long time. An efficient financial market is characterized by the complete reflection of all relevant information in the market prices, which means that permanent deviations from the fundamental justified valuation are impossible (Fama, 1970). Miller and Modigliani (1961) set up a first model to capture the idea of efficient stock markets. In this model the value of a stock should equal the rational expected, discounted value of all future dividends of the stock.

In the most recent financial crisis the validity of efficient financial markets was brought into question. Long time before the financial crisis Shiller (1981) recognized the phenomenon that especially stock price indices like the Standard and Poor's 500 Composite Stock Price Index are much too volatile to be explained by the traditional fundamental value model of Miller and Modigliani. Based on this observation Shiller develops a variance bound for efficient stock markets. He concludes the violation of the fundamental value model since the variance of

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the S&P 500 stock price index is five times higher as suggested by his variance bound.

Shiller (1981) initiates the discussion about excess volatility on financial markets. In the following years especially Flavin (1983), Marsh and Merton (1986) and Kleidon (1986) criticize Shiller's conclusion from his variance bound test due to undesired small finite sample properties in his test.

In response to Shiller (1981) and the rising critic against his variance bound test, Mankiw, Romer and Shapiro (1985) develop a modified variance bound relation which holds in finite samples. After some further refinements of their test, Mankiw et al. (1991) obtained more differentiated results with respect to the validity of the efficient market hypothesis. According to their results, there is a overall tendency to reject the hypothesis of efficient stock markets but this finding is less pronounced compared to the earlier study of Shiller (1991).

In this paper the framework of Mankiw et al. (1991) is applied to an updated data set in order to test the hypothesis of efficient stock markets.

The Original Variance Bound for Efficient Stock Markets

The idea of a variance bound in efficient financial stock markets bases upon the present value model by Miller and Modigliani (1961). This model can be characterized by the following two equations:

$$P_t^* = \sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^{i+1} D_{t+i} \qquad (1)$$

$$P_t = E_t P_t^*, \qquad (2)$$

where D_t denotes the dividend of a stock for period t and r is the expected return assumed to be constant. $E_t(\cdot)$ denotes the expectation conditional on information available at time t, especially the present and all past prices of the stock.

Therefore P_t^* is the unobservable ex post rational price of the stock and P_t is the rational forecast of P_t^* at time t. Equations (1) and (2) describe the fundamental value model, where the price of a stock equals the expected, discounted value of all future dividends. According to this model fluctuations of the market price of a stock should be fully explained by the emergence of new information about future dividends.

Shiller (1981) challenged the validity of the fundamental value model. The (ex post observable) series P_t^* appeared to be much smoother than the series of the actual market prices P_t. Shiller (1991) questioned that the disparity of the volatilities can be adequately explained by the mere emergence of new information. In order to analyze this issue empirically, Shiller derives the first variance bound for efficient stock markets. Equation (1) can be reformulated as

$$P_t^* \quad = \quad P_t + \epsilon_t \ , \tag{3}$$

where ϵ_t denotes the rational forecast error at time t with zero mean. Using the fact that under rational expectations ϵ_t has to be uncorrelated with all known information at time t , the variance of P_t^* simplifies to

$$Var(P_t^*) \quad = \quad Var(P_t) + Var(\epsilon_t) \tag{4}$$

and since $Var(\epsilon_t) \geq 0$, it follows that

$$Var(P_t^*) \geq Var(P_t). \tag{5}$$

Equation (5) states the simplest form of a variance bound in efficient stock markets and indicates that the variance of the ex post rational prices has to be at least as large as the variance of the market prices. Shiller (1991) compares the sample variance of P_t^* and P_t computed from the Standard and Poor's 500 Composite Stock Price Index for 1871–1979 and finds the variance of the market prices P_t to be five times higher than the one obtained from their rational ex post counterpart P_t^*. To ensure the variances of both series to be finite, Shiller assumes both series to be trend stationary. Obviously, this result questions the validity of the fundamental value model for stock markets.

In response to Shiller's work many authors criticize the assumptions and interpretation of his test. Flavin (1983) notes that both sample variances of P_t and P_t^* underestimate the true population variances. However the negative bias for the variance of P_t^* is larger, which means that the understimation for $Var(P_t^*)$ is stronger than for $Var(P_t)$. This negative bias results from replacing the unknown expectation by sample means.

Marsh and Merton (1986) challenge the entire interpretation of Shiller's results. According to them the violation of Shiller's variance bound in equation (5) does not necessarily imply a violation of the present value model. In contrast Shiller's method is a test of the joint hypothesis of efficient financial stock markets with constant return and a trend stationary dividend process. Following this argument a violation of the variance bound in (5) might occur due to a violation of the assumed constant return or the trend stationary dividend process even though the assumption of an efficient stock market is fulfilled.

Kleidon (1986) reveals a second weakness in Shiller's interpretation. He shows that from a single time series of the realized observations of P_t^* and P_t nothing can be concluded in terms of the validity of Shiller's variance bound. Kleidon explains this seemingly counterintuitive argument as follows. The variance bound (5) is based on repeated samples of the process $\{P_1^*, \dots, P_T^*\}$ because different realizations of future dividends result in different sequences of $\{P_1^*, \dots, P_T^*\}$. Shiller's variance bound implies that among all possible realizations the variance of P_t^* is expected to be larger than the variance of P_t. Accordingly the variance bound represents a cross-sectional relation between different states of an economy at time t and no relation between the time series variances.

The Modified Variance Bound Test

In response to Shiller's first trial, Mankiw et al. (1991) develop a modified test that is more accurate in finite samples. In order to derive their test statistic, some new definitions need to be introduced. Mankiw et al. (1991) define the ex post present value P_t^{*h} for the strategy of buying a stock at time t and holding it for h periods as

$$P_t^{*h} = \sum_{j=0}^{h-1} \left(\frac{1}{1+r}\right)^{j+1} D_{t+j} + \left(\frac{1}{1+r}\right)^{h} P_{t+h} \tag{6}$$

where D_{t+j} denotes the dividend in period $t+j$ and P_{t+h} is the market price in period $t+h$. Under the assumptions of the present value model it holds that

$$P_t = E_t P_t^{*h}. \tag{7}$$

The investment horizon h can be chosen as variable or constant. In the variable case, the investment horizon coincides with the end of the sample such that $h_t = T - t$. Alternatively h can be chosen as constant for every observation such that P_t^{*h} displays the ex post present value for the strategy of holding the stock until period $t+h$ and selling it for the market price. This new definition of P_t^{*h} is a basic component for the modified test. The derivation of this test is based on the ideas of Mankiw et al. (1985). Let P_t^0 be an arbitrary ("naive") forecast of the fundamental value of the stock:

$$P_t^0 = \sum_{k=0}^{\infty} \rho^{k+1} \widehat{D}_{t+k} \tag{8}$$

where $\widehat{D}_{t+k}$ denotes naive forecast for the dividend D_{t+k} at period t and ρ is the known (constant) discount factor. The naive forecast does not have to be rational (i.e. the forecast may neglect available information). It is important however that the market participants may have access to the naive forecast at period t, that is, the naive forecast is entailed in the investor's information set. In order to derive Mankiw et al.'s (1985) modified test the following identity serves as starting point:

$$P_t^{*h} - P_t^0 \quad = \quad (P_t^{*h} - P_t) + (P_t - P_t^0). \tag{9}$$

From equation (7) it follows that $P_t^{*h} - P_t$ displays the rational forecast error ϵ_t which is independent of any available information at period t. Therefore it holds that:

$$E_t[(P_t^{*h} - P_t)(P_t - P_t^0)] \quad = \quad 0. \tag{10}$$

Squaring equation (9), using expectations and substituting equation (10) yields:

$$E_t(P_t^{*h} - P_t^0)^2 = E_t(P_t^{*h} - P_t)^2 + E_t(P_t - P_t^0)^2. \tag{11}$$

Equation (11) will remain valid if the conditional expectations are normalized with any scaling variable W_t known at t. Equation (11) can be reformulated as

$$E_t\left(\frac{P_t^{*h} - P_t^0}{W_t}\right)^2 \quad = \quad E_t\left(\frac{P_t^{*h} - P_t}{W_t}\right)^2 + E_t\left(\frac{P_t - P_t^0}{W_t}\right)^2. \tag{12}$$

In a last step define

$$q_t = \left(\frac{P_t^{*h} - P_t^0}{W_t}\right)^2 - \left[\left(\frac{P_t^{*h} - P_t}{W_t}\right)^2 + \left(\frac{P_t - P_t^0}{W_t}\right)^2\right]. \qquad (13)$$

Equation (12) implies that $E_t(q_t) = 0$ holds. However, by the law of iterative expectation, this implies $E_{t-s}(q_t) = 0$ for all $s \geq 0$. Therefore a test of the hypothesis of efficient stock markets implies that

$$H_0 : \quad \alpha = 0, \qquad (14)$$

in the regression $q_t = \alpha + \varepsilon_t$, where ε_t is an error term with expectation zero. Since the expectation of q_t is zero for all t, the expectation of the mean $\overline{q}$ is zero as well. In order to construct a test statistic for the null hypothesis (14), asymptotically valid standard errors have to be constructed. Mankiw et al. (1991) stress the issue of autocorrelation in the errors. For constant holding periods h and under the assumptions of efficient markets q_t and q_{t-j} are correlated for $j < h$ but uncorrelated for $j \geq h$. In the case of variable holding periods h_t the correlation does not vanish after a fixed lag. To account for the autocorrelation in the error terms, Mankiw et al. (1991) use Newey-West standard errors which are asymptotically valid. In the case of a constant holding periods h, the truncation lag is set to $h - 1$ since the autocorrelation vanishes after this lag. In the case of variable holding periods the rule of thumb of Newey-West is chosen for the truncation lag.[1] The final test statistic is the square of the t-statistic $\widehat{\alpha}^2/\widehat{Var}(\widehat{\alpha})$, which is a two-sided Wald-statistic with a χ^2 distribution with one degree of freedom. Note that the estimated variance of $\widehat{\alpha}$ is calculated with the Newey-West standard errors.

[1] Newey-West's rule of thumb for the truncation lag for unknown autocorrelation is $P = int[4(T/100)^{\frac{2}{9}}]$.

Finally there are two more useful implications of Mankiw et al.'s (1985) test which can be derived from equation (12):

$$E\left(\frac{P_t^{*h} - P_t^0}{W_t}\right)^2 \quad \geq \quad E\left(\frac{P_t^{*h} - P_t}{W_t}\right)^2 \tag{15}$$

and

$$E\left(\frac{P_t^{*h} - P_t^0}{W_t}\right)^2 \geq E\left(\frac{P_t - P_t^0}{W_t}\right)^2 . \tag{16}$$

The first upper bound in (15) claims that the expected squared error with a naive forecast (P_t^0) should be at least as large as the expected squared error with the optimal forecast (P_t). The upper bound in (16) states that the volatility of P_t^* around P_t^0 should be at least as large as the volatility of P_t around P_t^0. If the null hypothesis is rejected both upper bound relations can be helpful to detect the source of rejection.

In contrast to Shiller's test, Mankiw et al. (1985) show that their upper bound relations in (15) and (16) are unbiased regardless of the sample size and the underlying dividend process. This is achieved by centering the variances around a naive forecast and not around the sample mean.

Empirical Analysis

In this section the modified test of Mankiw et al. (1991) is applied to real data in order to test the hypothesis of efficient stock markets. All time series are annual data from 1871 to 2011. The stock price series consists of data of the Standard and Poor's 500 Composite Price Index, where the price of a year is represented by the average of the daily closing prices for January. The dividend series consists of dividends per stock, added over 12 months and adjusted to the index for the fourth quarter of each year. Both series are converted to real units with the

Consumer Price Index. In order to conduct the test of efficient stock markets the naive forecast P_t^0 has to be specified. In a first version of the test Mankiw et al. (1991) specify P_t^0 derived from the no-change forecast of future dividends so that the expected dividends are identical to the last observed value (D_{t-1}). Therefore P_t^0 can be expressed as:

$$P_t^0 = \frac{1}{r} D_{t-1}. \tag{17}$$

Table 1 presents the results of the test using the naive forecast in (17) and constant returns of 5%, 6% and 7% for different holding periods. The Columns (ii), (iii) and (iv) display the sample mean of $[(P_t^{*h} - P_t^0)/P_t]$, $[(P_t^{*h} - P_t)/P_t]^2$ and $[P_t - P_t^0)/P_t]^2$. The scaling variable W_t is P_t in order to diminish the issue of heteroskedasticity since the variables are growing over time. Column (v) shows the result of the test statistic, which is the squared difference of column (ii) with the sum of column (iii) and (iv), divided by the variance of this difference. The variance is calculated with the Newey-West standard errors. Under the null hypothesis (14) the test statistic is asymptotically χ^2 distributed with one degree of freedom, implying a critical value of 3.84 for a significance level of 0.05. Column (vi) shows the respective p-values. The theory of efficient stock markets predicts that the entries in column (ii) should equal the sum of the entries in column (iii) and (iv) which is tested by the statistic in column (v). Furthermore the entries in column (ii) should be larger than the entries in column (iii) and (iv), as presented in the upper bound relations in (15) and (16).

In terms of the validity of inequality (15) Table 1 shows that the relation holds except for the case of $r = 5\%$ with variable holding periods $h = T - t$. Only in this case the naive forecast in (17) is a better forecast than the market price in terms of the forecast error variance. The inequality (16) is stable as well since column

(ii) almost always exceeds column (iv). The only exception is the case of $r = 7\%$ and variable holding periods. Accordingly the volatility of P_t^{*h} around the naive forecast in (17) is almost always larger than the volatility of P_t around the naive forecast. Therefore, the market prices are not excessively volatile around the naive forecast. The p-values of the test statistic for the hypothesis that column (ii) equals the sum of column (iii) and (iv) show a tendency for accepting the null hypothesis of efficient stock markets. Only the p-values of 5 and 10 year holding periods with an expected return of 5% yields a rejection of the null hypothesis at the 10% significance level. However the picture for the variable holding periods is different. The p-values imply a significant rejection of the null hypothesis for every expected return. The present value model with constant expected return is not supported for variable holding periods and the naive forecast defined in (17).

Table 2 shows the results of a similar test as before but with a different naive forecast. The alternative naive forecast consists of a thirty year moving average of the dividends and can be written as:

$$P_t^0 = \frac{1}{r}\left[\frac{1}{30}\sum_{i=1}^{30} D_{t-i}\right]. \tag{18}$$

Mankiw et al. (1991) choose this particular forecast to smooth the series P_t^0. The smoothed naive forecast should help to detect the excess volatility of the market prices. Furthermore, the scaling variable W_t is set to P_t^0, which is supposed to avoid a bias in the test results due to possible excess volatility in the series P_t used above. The test results in Table 2 indeed display excess volatility of the market prices around the naive forecast defined in (18). The values in column (iv) exceed the values in column (ii) for every holding period and every expected return. However the second upper bound relation always holds since column (ii) always exceeds column (iii). This implies that the market price P_t is a better forecast for the ex post rational price P_t^{*h} than the naive forecast in (18). The p-values imply acceptance of the null hypothesis for constant holding periods below 10 years for every expected return. However for the 10 year holding periods the null hypothesis has to be rejected. In the case of variable holding periods the rejections are much weaker compared to the tests presented in Table 1. In the case with $r = 5\%$ the null can be accepted at the 5% level. In general the test with the modified naive forecast displays a stronger tendency to accept the hypothesis of efficient stock markets except for the 10 year holding period.

Next we relax the assumption of constant expected returns. We follow Mankiw et al. (1991) and construct time varying expected return as the sum of a variable, riskless interest rate (r_t^*) and a constant risk premium (ϕ). Therefore the one period nominal discount factor is given by $\rho_t = 1/(1 + r_t^* + \phi)$. Under the hypothesis of efficient stock markets it holds that:

$$\begin{aligned} P_t &= E_t\left[\sum_{j=0}^{h-1} \rho_t^{j+1} D_{t+j} + \rho_t^h P_{t+h}\right] \\ &\equiv E_t P_t^{*h} \ . \end{aligned} \tag{19}$$

For the naive forecast Mankiw et al. (1991) assume that the dividends grow with the riskless interest rate. Therefore the naive forecast can be expressed as $P_t^0 = \frac{1}{\phi} D_{t-1}$. According to Mankiw et al. (1991) a test with variable discount factors is especially suitable if changes in the interest rates are considered as important drivers for market price volatility. In this case one would expect a rejection of the null hypothesis of efficient stock markets in tests with constant expected return.

Table 3 on page 11 shows the test results for different risk premiums $(4\%, 5\%, 6\%)$ and the same holding periods as in the tests before. The data for the riskless interest rate (r_t) are annual commercial paper rates of “triple A” ranked companies which are taken from the Federal Reserve Bank of St.Louis. Furthermore all data are in nominal terms now since the dynamic of the inflation is captured by the riskless interest rate. The results of the new tests in Table 3 do not support the validity of the present value model with variable discount rates. Even though the null hypothesis cannot be rejected for low holding periods of one and two years, there is a strong tendency towards rejection for longer holding periods. Especially the variable holding periods lead to strong rejections. The upper bound for the volatility of the market prices is, like in the case of constant returns, almost always met. The upper bound of the forecast error variance holds for constant holding periods only. In general the test with variable discount rates exhibits a stronger tendency for rejecting the null hypothesis compared to the test with constant returns.

Table 4 shows the test results for variable discount rates based on the smoothed naive forecast $P_t^0 = \frac{1}{\phi}\left[\frac{1}{30}\sum_{i=1}^{30} D_{t-i}\right]$. The scaling factor W_t is again P_t^0. The test results tend to be similar to those of the analog test with constant returns. The upper bound of the volatility is almost always violated, whereas the upper bound for the forecast error variance is always met. The null hypothesis is accepted more often with the smoother naive forecast compared to the test before. The results of the tests with variable discount rate show that variation in the interest rates does not seem to play an important role for the volatility of the stock market prices. The tests do not yield more evidence in favor of the null hypothesis but tend to stronger rejections.

Conclusion

In this paper two different variance bounds tests were considered. Shiller's (1981) approach leads to a strong rejection of the hypothesis of efficient stock markets. However, due to the undesired small sample properties and the sensitivity in terms of the underlying dividend process, his results are not reliable. The results of the modified variance bound test of Mankiw et al. (1991) yields a more differentiated picture. Using this test, the hypothesis of efficient stock markets cannot be generally accepted, however it cannot be definitely rejected neither. For low holding periods there is a tendency for accepting the null hypothesis. In contrast, especially variable holding periods lead to a clear rejection. The degree of significance of the rejections depends on the choice of the naive forecast. The smoothed naive forecast yields weaker rejections of the null hypothesis. Furthermore it was shown that variation in the interest rates is not a significant driver of price volatility on stock markets since the test with variable discount rates leads even to stronger rejections.

To sum up, the modified test of Mankiw et al (1991) with an updated sample neither delivers clear evidence against the fundamental value model nor does it generally support the hypothesis .

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Appendix

Tables

(i)	(ii)		(iii)		iv)	(v)	(vi)
h	$E\left[\frac{P_t^{*h}-P_t^0}{P_t}\right]^2$	$=$	$E\left[\frac{P_t^{*h}-P_t}{P_t}\right]^2$	$+$	$E\left[\frac{P_t-P_t^0}{P_t}\right]^2$	χ^2	p-value
					$r = 5\%$		
1	0.1401		0.0295		0.1203	1.1560	0.2823
2	0.1542		0.0628		0.1186	1.8879	0.1694
5	0.2038		0.1583		0.1146	3.9917	0.0457
10	0.2993		0.2933		0.1015	3.2229	0.0726
$T-t$	0.3119		0.3450		0.1224	10.7373	0.0010
					$r = 6\%$		
1	0.1670		0.0284		0.1420	0.1213	0.7276
2	0.1854		0.0588		0.1398	0.3596	0.5487
5	0.2345		0.1358		0.1344	0.6263	0.4287
10	0.3005		0.2192		0.1191	0.7466	0.3876
$T-t$	0.1886		0.1364		0.1445	7.9729	0.0047
					$r = 7\%$		
1	0.2113		0.0276		0.1880	0.1463	0.7021
2	0.2274		0.0556		0.1857	0.3009	0.5833
5	0.2614		0.1193		0.1798	0.5654	0.4521
10	0.2893		0.1707		0.1640	1.3156	0.2514
$T-t$	0.1235		0.0921		0.1906	15.8151	0.0001

Note: column (i): holding periods; column (ii)-(iv): sample estimator (sample means) of the expectation from (12, weighted with the market price; column (v): $\chi^2(1)$ test statistic for the hypothesis, that column (ii) equals the sum of (iii) and (iv) ; column (vi): p-values of the test statistic. The naive forecast is defined in (17).

Table 1: Test with a naive forecast, constant expected return (r)

(i)	(ii)		(iii)		(iv)	(v)	(vi)
h	$E\left[\frac{P_t^{*h}-P_t^0}{P_t^0}\right]^2$	$=$	$E\left[\frac{P_t^{*h}-P_t}{P_t^0}\right]^2$	$+$	$E\left[\frac{P_t-P_t^0}{P_t^0}\right]^2$	χ^2	p-value
					$r = 5\%$		
1	1.3686		0.1058		1.3911	0.8206	0.3650
2	1.3167		0.2347		1.3740	0.6623	0.4158
5	1.1597		0.4299		1.2506	0.8374	0.3601
10	0.9174		0.6797		0.9729	11.7948	0.0006
$T-t$	0.9960		0.4948		1.4181	2.8938	0.0889
					$r = 6\%$		
1	2.2756		0.1514		2.3654	1.1434	0.2849
2	2.1420		0.3340		2.3358	0.8392	0.3596
5	1.7711		0.6000		2.1363	0.9293	0.3350
10	1.2679		0.9023		1.6894	10.2238	0.0014
$T-t$	1.1505		0.6311		2.4104	4.1781	0.0409
					$r = 7\%$		
1	3.4241		0.2060		3.6384	1.5866	0.2078
2	3.1554		0.4541		3.5933	1.0804	0.2986
5	2.4531		0.8164		3.3005	1.0698	0.3010
10	1.5932		1.2031		2.6466	9.9416	0.0016
$T-t$	1.2360		1.0670		3.7056	5.9329	0.0149

Note: See Table 1. The naive forecast is defined in (18).

Table 2: Test with naive forecasts based on smoothed dividends, constant expected return

(i)	(ii)		(iii)		(iv)	(v)	(vi)
h	$E\left[\frac{P_t^{*h}-P_t^0}{P_t}\right]^2$	$=$	$E\left[\frac{P_t^{*h}-P_t}{P_t}\right]^2$	$+$	$E\left[\frac{P_t-P_t^0}{P_t}\right]^2$	χ^2	p-value
					$\phi = 4\%$		
1	0.1944		0.0293		0.1820	2.0742	0.1498
2	0.1962		0.0616		0.1815	4.2748	0.0387
5	0.1993		0.1353		0.1811	5.9018	0.0151
10	0.2653		0.2508		0.1743	1.6808	0.1948
$T-t$	0.3150		0.4310		0.1829	8.4567	0.0036
					$\phi = 5\%$		
1	0.1401		0.0286		0.1206	1.0703	0.3009
2	0.1494		0.0585		0.1189	2.5016	0.1137
5	0.1633		0.1200		0.1149	4.4654	0.0346
10	0.2178		0.1995		0.1017	3.9402	0.0471
$T-t$	0.2023		0.2125		0.1227	12.7049	0.0004
					$\phi = 6\%$		
1	0.1602		0.0280		0.1422	1.1447	0.2847
2	0.1688		0.0562		0.1400	1.8632	0.1723
5	0.1743		0.1096		0.1346	3.3706	0.0664
10	0.2018		0.1679		0.1193	4.9080	0.0267
$T-t$	0.1410		0.1549		0.1448	20.8802	0.0000

Note: See Table 1. The naive forecast is $P_t^0 = \frac{1}{\phi}D_{t-1}$. The data are in nominal terms.

Table 3: Test using naive forecast with recent dividends and variable interest rates

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(i)	(ii)		(iii)		(iv)	(v)	(vi)
h	$E\left[\frac{P_t^{*h}-P_t^0}{P_t^0}\right]^2$	$=$	$E\left[\frac{P_t^{*h}-P_t}{P_t^0}\right]^2$	$+$	$E\left[\frac{P_t-P_t^0}{P_t^0}\right]^2$	χ^2	p-value
					$\phi = 4\%$		
1	2.1453		0.1325		2.1594	0.5018	0.4787
2	2.1070		0.2954		2.1419	0.3577	0.5498
5	1.9917		0.5315		1.9918	0.5275	0.4676
10	1.7463		0.8426		1.6128	11.2890	0.0008
$T-t$	1.7607		0.6465		2.1874	2.1543	0.1422
					$\phi = 5\%$		
1	3.8768		0.2056		3.9888	0.8075	0.3689
2	3.7248		0.4556		3.9551	0.5225	0.4698
5	3.3027		0.8007		3.6903	0.7522	0.3858
10	2.6312		1.2010		3.0285	13.0424	0.0003
$T-t$	2.3323		0.8698		4.0405	3.5394	0.0599
					$\phi = 6\%$		
1	6.1238		0.2959		6.4341	1.2280	0.2678
2	5.7631		0.6545		6.3794	0.7409	0.3894
5	4.8122		1.1483		5.9690	1.0024	0.3167
10	3.5014		1.6839		4.9491	13.7785	0.0002
$T-t$	2.8715		1.4906		6.5161	5.1177	0.0237

Note: See Table 1. The naive forecast is $P_t^0 = \frac{1}{\phi}\left[\frac{1}{30}\sum_i^{30} D_{t-i}\right]$. The data are in nominal terms.

Table 4: Test with smoothed naive forecast and variable interest rates